Optimal Scaling Auctions:
A Consumer Theory Deconstruction

Ernesto Rivera Mora [1] Allan Hernandez [2]

[1]University of Colorado Boulder

[2]University of Queensland

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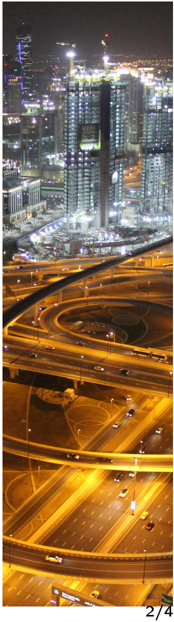
Joint Work

Joint work with **Allan Hernandez**.

University of Queensland

Public Infrastructure Procurement



Departments of Transportation:

- Assign infrastructure projects for (bridges, highways) to private construction firms.
- Two types of auctions used in the US:
  1. Cash auctions -> each bid is a price
  2. Scaling auctions -> each bid is a vector of prices

What are Cash Auctions?

- Each bidder (construction firm) submits a price for the project.
- The bidder with the lowest price wins.
  - e.g. Firm A bids $\$20M$ and Firm B bids $\$25M$

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    ⇒ Firm A wins and receives $\$20M$ to build the bridge
- **Issue:** Building a bridge is risky!
    - They require multiple inputs/tasks (concrete, steel, paint, etc)
    - Hard to predict input quantities
    - Extra inputs are not covered by the contract price.

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Current practice: $w_t = \mathbb{E}[q_t]$

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2. Which *scoring rule* minimizes the auctioneer's payment?
3. In which environments *scaling auctions* dominate *cash auctions*?
4. What are the outcomes of a *hybrid auction*?
5. What are the effects of allowing bidders gather information?

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3. Scaling auctions dominate cash auctions iff:

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4. An **Hybrid auction** dominates scaling and cash auctions.
5. Allowing bidders to acquire information benefits (only) the auctioneer.

Methodology: Consumer Theory Deconstruction

From **game theory** to **consumer theory**

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1. Define the **auxiliary auction**: simple game easy to analyze.
2. Each equilibrium of a scaling auction corresponds to an equilibrium of an **auxiliary auction**.
3. **Utility equivalence**: Bidders are indifferent among scoring rules.
4. Bidders' equilibrium solves a **Hicksian demand** of some consumer problem
    - **Law of demand** and **cost-minimization duality**

Related Literature

- **Empirical Regularities of Scaling Auctions:**

  Luo and Takahashi (2022); Bolotny and Vasserman (2023)

- **Scoring Auctions:**

  Che (1993); Asker Cantillon (2008, 2010); Hanazono (2024); Awaya et al. (2022)

- **Auctions with Risk-Averse Bidders:**

  Riley and Samuelson (1981); Maskin and Riley (1984); Matthews (1987)

- **Auctions with Contingent Payments:**

  DeMarzo, Kremer, and Skrzypacz (2005); Abhishek, Hajek and Williams (2015); Fioriti and Hernandez-Chanto (2022)

Model

The Project and its Tasks

- $\mathcal{I} = \{1, 2, \ldots, I\}$ risk-averse bidders compete for a project.
- Project's tasks: $\mathcal{T} = \{1, 2, \ldots, T\}$.
- The project requires unknown input quantities $\boldsymbol{q} = (q_t)_{t \in \mathcal{T}}$.
- Distribution of $\boldsymbol{q}$ given by probability measure $\mu \in \Delta(\mathbb{R}^T)$.

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Bidders

Bidders maximize expected utility given:

- CARA utility function $u(\pi_i) := 1 - \exp\left(-\gamma \cdot \pi_i\right)$
- Marginal costs: $c_{i,t} = c_t \rightarrow$ Common to bidders.
- Fixed costs k_i:
    - Include opportunity costs.
    - Bidder i's private information.
    - Drawn independently from distribution F with density f.

Auctioneer

The auctioneer (Department of Transportation) is risk-neutral:

- Minimizes project's expected payment
- Chooses either a scaling auction or a cash auction

Scaling Auctions

Definitions and Notation

- A **scaling auction** is characterized by the pair $\mathcal{A} = (\boldsymbol{w}, \mathcal{B})$.
  - Weights: $\boldsymbol{w} = (w_t)_{t \in \mathcal{T}} \in \mathbb{R}_+^T$.
  - Set of admissible bids: $\mathcal{B} \subseteq \mathbb{R}^T$.
- Each vector of bids $\boldsymbol{b}_i = (b_{i,t})_{t \in \mathcal{T}} \in \mathcal{B}$ induces a score:

$$s_i = \boldsymbol{w} \cdot \boldsymbol{b}_i = \sum_t w_t \, b_{i,t}$$

- Lowest score wins the auction.
- Ex-post payment: $\boldsymbol{q} \cdot \boldsymbol{b}_i$.
- Bidder's ex-post monetary payoff: $\pi_i = \boldsymbol{q} \cdot (\boldsymbol{b}_i - \boldsymbol{c}) - k_i$.

Interim and ex-ante expected utility

- Two layers of uncertainty:
  - Risk inherent to the project conditional on winning.
  - Risk of losing the auction.
- Expected utility of winning for k_i-bidder who bids $\boldsymbol{b}_i$

$$U(\boldsymbol{b}_i \mid k_i) := \mathbb{E}\Big[u\big(\boldsymbol{q} \cdot (\boldsymbol{b}_i - \boldsymbol{c}) - k_i\big)\Big].$$

- The expected utility of a k_i-bidder who bids $\boldsymbol{b}_i$

$$U(\boldsymbol{b}_i \mid k_i) \times \underbrace{\mathbb{P}\big[\boldsymbol{b}_i \cdot \boldsymbol{w} < s_j \text{ for all } j \neq i\big]}_{\text{Probability of winning}}.$$

Properties of $U(\cdot \mid k_i)$

1. **Indifference curves are convex**
   - $U(\cdot \mid k_i)$ is strictly convex
   - Portfolio problem: Intermediate $\boldsymbol{b}_i$'s are less risky than extreme $\boldsymbol{b}_i$'s.
2. **Identical indifference curves**
   - $U(\cdot \mid k_i)$ induces the same indifference curves as $U(\cdot \mid k_i')$
   - CARA preferences ⇒ No income effects

Characterizing Equilibria

To characterize equilibria we divide the problem into two:

1. **The Inner Loop:** Find optimal bid vector $\boldsymbol{b}_i$ for a target score s_i
   - "The portfolio problem"
   - Maximize $U(\cdot \mid k_i)$ conditional on a score s_i
2. **The Outer Loop:** Find score s_i that maximizes expected utility
   - "The auction problem"

“Inner Loop”: The Portfolio Problem

Fix $\mathcal{A} = (\boldsymbol{w}, \mathcal{B})$ and write $\mathcal{B}[s_i \mid \boldsymbol{w}] := \left\{\boldsymbol{b}_i \in \mathcal{B} : \boldsymbol{b}_i \cdot \boldsymbol{w} = s_i\right\}$ for the bids inducing score s_i

$$V(s_i \mid k_i, \mathcal{A}) := \max_{\boldsymbol{b}_i \in \mathcal{B}[s_i | \boldsymbol{w}]} U\left(\boldsymbol{b}_i \mid k_i\right),$$

$$\boldsymbol{b}_i(s_i \mid k_i, \mathcal{A}) \in \arg \max_{\boldsymbol{b}_i \in \mathcal{B}[s_i | \boldsymbol{w}]} U\left(\boldsymbol{b}_i \mid k_i\right).$$

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- We call $\boldsymbol{b}_i(\cdot \mid \cdot)$ the **Marshallian bidding demand**.
    - Identical utility curves ⇒ $\boldsymbol{b}_i(s_i \mid k_i, \mathcal{A}) = \boldsymbol{b}_i(s_i \mid k_i', \mathcal{A})$

"Outer Loop": The Auction Problem

Fix a monotone score strategy: $\varphi : [\underline{k}, \bar{k}] \to \mathbb{R}$.

Bidder's choice of scores

Given φ, bidder i chooses score s_i to maximize:

$$\mathrm{EU}(s_i \mid k_i, \mathcal{A}) := \underbrace{V\left(s_i \mid k_i, \mathcal{A}\right)}_{\text{Value of winning}} \cdot \underbrace{\left[1 - F\left(\varphi^{-1}(s_i)\right)\right]^{I-1}}_{\text{Probability of winning}}$$

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Equilibrium

φ is a **Bayesian equilibrium** of $\mathcal{A}$ if

$$\mathrm{EU}(\varphi(k_i) \mid k_i, \mathcal{A}) \geq \mathrm{EU}(s_i \mid k_i, \mathcal{A}) \quad \forall s_i$$

Rich Auctions

This talk: Focus on auctions with large bidding sets

Definition

Call $\mathcal{A} = (\boldsymbol{w}, \mathcal{B})$ ***rich*** *if the bidding set* $\mathcal{B}$ *is sufficiently large so that all bidders have incentives to participate.*

Utility Equivalence and Hicksian Decomposition

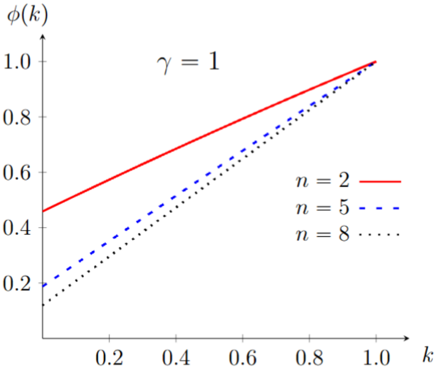
The Auxiliary Auction

Auxiliary Auction: a simple game where

- Each bidder bids $r_i \in \mathbb{R}$.
- The bidder who bids the lowest r_i wins and obtains r_i.
- Fixed costs $k_i \sim F$, no marginal costs
- Utility of winning bidder with cost k_i and bid r_i: $u(r_i - k_i)$.

Equilibrium of the Auxiliary Auction

$F \sim unif[0,1]$



$$\psi'(k_i) = \frac{(I-1)f(k_i)u(\psi(k_i)-k_i)}{(1-F(k_i))u'(\psi(k_i)-k_i)} \quad \text{with} \quad \psi(\bar{k}) = \bar{k}$$

Equivalence between scaling auction and auxiliary auction

Theorem: Equivalence

Each equilibrium of a scaling auction induces the same utility for the bidders as the equilibrium of the ***auxiliary auction****.*

Equivalence between scaling auction and auxiliary auction

- $\psi : [\underline{k}, \bar{k}] \to \mathbb{R}$ equilibrium of the **auxiliary auction**
- $\mathrm{CE}(s_i \mid \mathcal{A})$: Certainty equivalent of lottery $\boldsymbol{q} \cdot (\boldsymbol{b}_i(s_i) - \boldsymbol{c})$

Theorem: Equivalence

$\varphi : [\underline{k}, \bar{k}] \to \mathbb{R}$ is a symmetric equilibrium of $\mathcal{A}$ if and only if:

$$\mathrm{CE}(\varphi(k_i) \mid \mathcal{A}) = \psi(k_i), \text{ and}$$

$$V\big(\varphi(k_i) \mid k_i, \mathcal{A}\big) = u\big(\psi(k_i) - k_i\big).$$

Equilibrium equivalence

Corollary: Utility Equivalence

All scaling auctions induce the same utility on the bidders.

Equilibrium equivalence

Corollary: Utility Equivalence

If $\varphi : [\underline{k}, \bar{k}] \to \mathbb{R}$ is a Bayesian equilibrium of $\mathcal{A}$ and $\varphi' : [\underline{k}, \bar{k}] \to \mathbb{R}$ is a Bayesian equilibrium of $\mathcal{A}'$, then

$$V\big(\varphi(k_i) \mid k_i, \mathcal{A}\big) = V\big(\varphi'(k_i) \mid k_i, \mathcal{A}'\big)$$

Optimal Bidding as a Hicksian Demand

- Utility equivalence → bidders' optimal bids can be obtained as the solution of a score minimization problem.

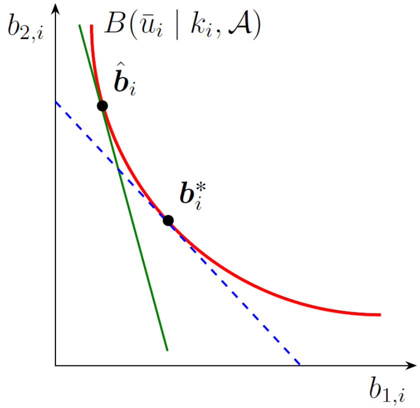
Score Minimization Problem

Write $B(\bar{u}_i \mid k_i) := \{\boldsymbol{b}_i \in \mathcal{B} : U(\boldsymbol{b}_i \mid k_i) = \bar{u}_i\}$

$$\boldsymbol{h}_i(\bar{u}_i \mid k_i, \mathcal{A}) := \arg \min_{\boldsymbol{b} \in B(\bar{u}_i \mid k_i, \mathcal{A})} \boldsymbol{b} \cdot \boldsymbol{w}$$

The solution $h_i(\bar{u}_i \mid k_i, \mathcal{A})$ is called the **Hicksian bidding demand**.

Duality



If $\bar{u}_i$ is the utility of k_i, then $\boldsymbol{b}_i(\varphi(k_i) \mid k_i, \mathcal{A}) = \boldsymbol{h}_i(\bar{u}_i \mid k_i, \mathcal{A})$.

Law of Demand

A change in the vector $\boldsymbol{w}$ has two effects on the equilibrium bid:

1. A null *income effect*:
    - A change in $\boldsymbol{w}$ changes the score s_i of each i
    - The competitive nature of the auction leads zero effect on utility.
2. A pure *substitution effect*:
    - Law of demand: $\nearrow w_t \quad \Rightarrow \quad \searrow b_{it}$
    - Consistent with empirical literature: “bid skewing”

Optimal Scaling Auctions

Theorem: Optimal Scoring Rule

If $\mathcal{B} = \mathbb{R}^T$ and $\boldsymbol{w} = \lambda \cdot \mathbb{E}[\boldsymbol{q}]$ for some $\lambda > 0$, then $\mathcal{A} = (\boldsymbol{w}, \mathcal{B})$ is optimal.

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- The auctioneer "delegates" his minimization problem.

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- The auctioneer "delegates" his minimization problem.

Remark: $\mathcal{B}$ is required to include negative bids

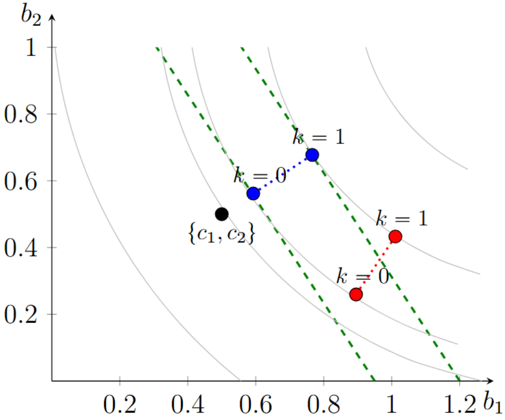
Example 1

- Two competing bidders.
- Fixed costs independently drawn from $Unif[0,1]$.
- Two tasks. Marginal cost of each task is $c_t = \frac{1}{2}$.
- CARA coefficient is $\gamma = 3$.
- The distribution of quantities is Gaussian parametrized by:

$$(\bar{q}_1, \bar{q}_2) = (3,2) \text{ and } \sigma_1^2 = \sigma_2^2 = 1, \quad \sigma_{1,2} = 0$$

Example 1

- Optimal auction: $\boldsymbol{w} = (3, 2)$, $\mathcal{B} = \mathbb{R}^T$
- Suboptimal auction: $\boldsymbol{w}' = (2, 3)$, $\mathcal{B} = \mathbb{R}^T$



Example 2: Optimality of Negative Bids

- Two tasks, two possibilities: $(q_1, q_2) = (4, 0)$ and $(q_1, q_2) = (6, 10)$
    - Half probability each
- Optimal auction: $\boldsymbol{w} = (5, 5), \mathcal{B} = \mathbb{R}^T$
- Sub-optimal auction: $\boldsymbol{w} = (5, 5), \mathcal{B} = \mathbb{R}_+^T$

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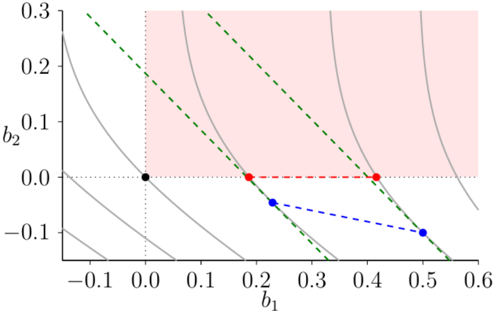

Cash Auctions

Strategies in the Cash Auctions and Auxiliary Auctions

Theorem: Utility Equivalence

In each rich **scaling auction** $\mathcal{A}$, as well as in the **auxiliary auction** and the **cash auction**, a k_i-bidder expects the same equilibrium utility.

Auctioneer's Cost Comparison

Bidder's utility equivalence + Risk-neutral auctioneer:

Minimizing expected payment ⇔ Minimizing risk bidders face

- Risk of Cash auction: $r_i - \boldsymbol{c} \cdot \boldsymbol{q} - k_i$
- Risk of Scaling auction: $(\boldsymbol{b}_i - \boldsymbol{c}) \cdot \boldsymbol{q} - k_i$

Auctioneer's Cost Comparison

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Minimizing expected payment ⇔ Minimizing risk bidders face

- Risk of Cash auction: $r_i - \boldsymbol{c} \cdot \boldsymbol{q} - k_i$
    - Independent of fixed costs and F
- Risk of Scaling auction: $(\boldsymbol{b}_i - \boldsymbol{c}) \cdot \boldsymbol{q} - k_i$

Auctioneer's Cost Comparison

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Minimizing expected payment ⇔ Minimizing risk bidders face

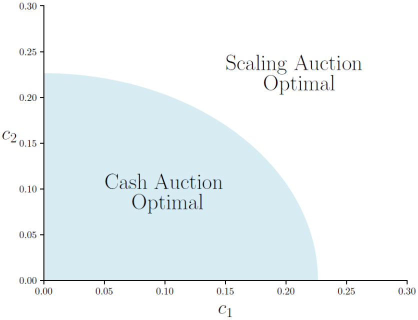
- Risk of Cash auction: $r_i - \boldsymbol{c} \cdot \boldsymbol{q} - k_i$
- Risk of Scaling auction: $(\boldsymbol{b}_i - \boldsymbol{c}) \cdot \boldsymbol{q} - k_i$
  - Markups $\boldsymbol{b}_i - \boldsymbol{c}$ are not affected by changes in $\boldsymbol{c}$:
    - Utility equivalence implies $\nearrow \boldsymbol{b}_i = \nearrow \boldsymbol{c}$
  - Risk is independent of marginal costs

Cash Auctions vs Scaling Auctions

Theorem:

The scaling auction dominates the cash auction iff marginal costs are sufficiently low:

$$\underbrace{\text{Risk}\,(\text{cash auction})}_{\text{depends on } c} \geq \underbrace{\text{Risk}\,(\text{scaling auction})}_{\text{independent of } c}$$



Example 1

Hybrid Auctions

Hybrid auctions

Scaling Auction with endogenous lump sum payment

- $b_{0,i}$: bid for lump sum payment.
- The auctioneer selects $\boldsymbol{w} = (w_0, \boldsymbol{w}) \in \mathbb{R}_+^{T+1}$ and $\mathcal{B} \subset \mathbb{R}^{T+1}$.
- $w_0 \in \mathbb{R}_+$ weight of the lump sum payment
- Extended set of tasks $\mathcal{T} = \{0, \ldots, T\}$.
- Quantities: $\boldsymbol{q} = (q_t : t \in \mathcal{T})$ with $q_0 = 1$
    - Beliefs: $\mu \in \Delta(\mathbb{R}^{T+1})$

Optimality of Hybrid Auctions

Theorem:

- Fix a hybrid scaling auction $\mathcal{A} = (\boldsymbol{w}, \mathcal{B})$.
- If $\boldsymbol{w} = (1, \mathbb{E}[q_1], ..., \mathbb{E}[q_T])$ and $\mathbb{R}_+^T \subseteq \mathcal{B}$, then:
  - $\mathcal{A}$ provides full insurance.
  - $\mathcal{A}$ dominates cash auctions and each pure scaling auction.

Value of Information

Value of Information

- All bidders observe a public signal
- Bidders' utility equivalence still holds → two opposing effects:
  - Informativeness effect:

    Bidders have more accurate valuations of bids.
  - Competitiveness effect:

    Bidders bid more aggressively.
- Auctioneer absorbs all benefits

Conclusion

- We analyze scaling auctions using a consumer theory approach:
  - Equilibrium → Hicksian Demand.
- Policy advice:
  - What are they doing right? $\boldsymbol{w} = \lambda \mathbb{E}[\boldsymbol{q}]$
  - What are they wrong? Forbidding penny bids
  - Optimal: Hybrid auctions with $\underline{\text{endogenous}}$ lump-sum
  - Allow bidders to obtain information
  - Get better information about $\mathbb{E}[q_t]/\mathbb{E}[q_{t'}]$
    - Information about substitutes rather than complements

Thank you!

Complementary Slides

Restrictions on the Bidding Set

- Bidding restrictions can decrease expected payment at a cost of failing the project with some probability.

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- Bidding restrictions can decrease expected payment at a cost of failing the project with some probability.
  - D: social benefit of the project.
  - k^* is the bidders' maximum cost that allows a bidder to participate in $\mathcal{A}$.
  - Restricting $\mathcal{B}$ lowers bids but lowers k^* raising the chance of failing the project.

Examples of Tasks Used in SAs

Most Common Tasks in SAs According to the FDoT

Mobilization
Maintenance of Traffic
Work Zone Sign
Temporary Barricade
Advance Warning / Arrow Board
High Intensity Flashing Lights
Temporary Retro-reflective Pavement Marker
Portable Changeable Message Sign
Clearing & Grubbing
Painted Pavement Markings

Empirical Regularities: FDOT vs. MDoT

- Luo & Takahashi, 2022 (Flordia DoT)
  - Uses SAs for high-risk and CAs for low-risk projects.
  - Skewed bids increase probability of winning.
  - Bid dispersion: Higher in CAs → Hedging behavior.
  - Bidder participation: Fewer in CAs → Insurance role.
- Bolotnyy & Vasserman, 2023 (Massachusetts DoT)
  - Sellers skew bids, no penny bids (risk aversion).
  - Removing uncertainty cuts spending by 14.5%, reflecting a significant risk premium.

Indirect Utility and Certainty Equivalent

- Indirect utility, conditional on winning

$$V(s_i \mid k_i, \mathcal{A}) \equiv U\big(\boldsymbol{b}_i(s_i \mid \mathcal{A}) \mid k_i, \mathcal{A})\big)\big].$$

- Certainty Equivalent

$$u\big(\mathrm{CE}(s_i \mid \mathcal{A})\big) = \mathbb{E}_\mu\big[u\big(\boldsymbol{q} \cdot (\boldsymbol{b}_i(s_i \mid \mathcal{A}) - \boldsymbol{c})\big)\big].$$

- Equivalence in utilities

$$u\big(\mathrm{CE}(s_i \mid \mathcal{A}) - k_i\big) = \mathbb{E}_\mu\big[u\big(\boldsymbol{q} \cdot (\boldsymbol{b}_i(s_i \mid \mathcal{A}) - \boldsymbol{c}) - k_i\big)\big] = V\big(s_i \mid k_i, \mathcal{A}\big).$$

- All bidders' types agree about the certain equivalent value $\mathrm{CE}(s_i \mid \mathcal{A})$.